

APPLICATION NOTE AN-002_EN

Measuring Principals – Decay Statistics – Test Planning

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The determination of the activity concentration of Radon is always a radiometric measurement, meaning a counting experiment. This causes a number of specific circumstances which have to take in consideration by the one who is carrying out this task. Only the knowledge of those particularities allows the correct set-up of a test and avoids misinterpretations of the achieved results.

Please read carefully the next chapters “Theory of Operation” and “Statistical Error” to become familiar with this kind of radiometric measurements.

Theory of Operation

The Radon ($Rn-222$) gas concentration will be measured by the short living daughter products, generated by the Radon decay inside a measurement chamber. Directly after the decay, the remaining $Po-218$ nuclei becomes charged positively for a short period, because some shell electrons are scattered away by the emitted alpha particle. Those ions are collected by the electrical field forces on the surface of a semiconductor detector. The number of collected $Po-218$ ions is proportional to the Radon gas concentration inside the chamber.

$Po-218$ itself decays with a half life time of only 3.05 Minutes and about 50% (particles emitted towards the detector surface) of all decays will be registered by the detector.

The equilibrium between the Radon decay rate and $Po-218$ detector activity is given after about 5 half life times, say 15 Minutes. This time span defines the minimum achievable response time to a Radon concentration step.

Now, the decay chain is continued by the both beta emitters $Pb-214$ and $Bi-214$ followed by another alpha emitter, the $Po-214$. That means, each $Po-218$ decay causes one more detectable decay by the $Po-214$ which is delayed about 3 hours because of the superposed half life times of those nuclides. The emission energies of $Po-218$ and $Po-214$ are different and therefore it is possible to separate both nuclides from each other by alpha spectroscopy.

Spectroscopic working monitors offer two calculation modes for the Radon concentration, one (Slow) includes both, $Po-218$ and $Po-214$ decays and the other one includes $Po-218$ only (Fast). The advantage of the “Fast” mode is the quick response to concentration changes while the “Slow” mode gives a sensitivity twice as high compared with the fast mode. The higher sensitivity reduces the statistical error of a measurement which depends on the number of counted decay events only. The user should select the calculation mode carefully with respect to the application specific requirements (see next chapter).

In case of Thoron ($Rn-220$), the direct daughter product $Po-216$ (which also underlies the ionisation process) is used to calculate the Thoron activity concentration. The half life of $Po-216$ is less than 1s and therefore the equilibrium state between gas concentration and collected activity on the detector is present immediately.

The half life of the Po-216 decay products Pb-212 (beta) and Pb-212/Bi-212 (alpha) are too long to use them for Thoron measurement. The single nuclides of the Thoron decay chain will also be separated by alpha spectroscopy.

Statistical Error (for non mathematicians)

The radioactive decay is a statistical process. That means, even if the Radon concentration is constant over the time, the number of decays N counted within several intervals of the same period will be different. N will vary around the mean value of all considered intervals. Considering an infinite number of intervals would lead to an average which one indicates the “true” result of N . For a single interval, the value of N will be either below or above the “true” value. This observed deviation is covered by the term “Statistical Error”.

Therefore, each serious measurement contains beside the calculated Radon value the error band for a stated confidence interval. The commonly used confidence intervals are 1, 2 or 3 Sigma (σ) which refer to a likelihood of 68.3%, 95.45% and 99.73%.

For example, the correct interpretation of a measured Radon concentration of 780 Bq/m³ with a statistical 1 σ error of $\pm 15\%$ is:

The real “true” Radon concentration lies with a likelihood of 68.3% within the range from 663 Bq/m³ (780 Bq/m³ - 15%) to 897 Bq/m³ (780 Bq/m³ + 15%).

Error Prediction

The relative statistical error E for a chosen confidence interval of k -Sigma can be predicted from the number of detected counts N by the equation:

$$E[\%] = 100\% * k * \sigma(N) / N$$

The simple consequence is: The higher the number of counts the higher is the accuracy of the measurement. From the opposite point of view one could ask: How many counts I have to detect to achieve a predefined uncertainty?

Two items will affect the number of counted decays: The sensitivity of the instrument at the one hand side and the time period used for counting process (integration interval) on the other hand.

While the sensitivity is an instrument specific constant, the integration interval may be expanded to the maximum acceptable value for the desired time resolution of a measurement series.

The relationship between the measured Radon concentration C_{Rn} and the number of counts N within an integration interval T is:

$$C_{Rn} = N / (T * S)$$

whereby S represents the Sensitivity of the instrument, given in the unit [cts/(min*kBq/m³)].

The sensitivity using the slow mode is double as high as in the fast mode (see chapter “Theory of Operation”) and whenever the required response time is more than 2 hours the slow mode should be selected.

For the following examples a fast mode sensitivity of 4 cts/(min*kBq/m³) shall be assumed while the slow mode sensitivity shall be 8 cts/(min*kBq/m³).

The first question could be: What an integration interval T have to set to get a statistical uncertainty less than 10% at a confidence level of 1 σ if the expected Radon concentration is 200 Bq/m³?

A 10% error of 100 counts ($100\% \cdot 1 \cdot \frac{100}{100} = 10\%$). Using the fast mode, the integration interval can be calculated by

$$T(\text{fast}) = N / (C_{Rn} \cdot S) = 100 \text{ cts} / (0.2 \text{ kBq/m}^3 \cdot 4 \text{ cts}/(\text{min} \cdot \text{kBq/m}^3)) = 125 \text{ min.}$$

Because the required interval is longer than 2 hours, the slow mode is the better choice, leading to the following result:

$$T(\text{slow}) = N / (C_{Rn} \cdot S) = 100 \text{ cts} / (0.2 \text{ kBq/m}^3 \cdot 8 \text{ cts}/(\text{min} \cdot \text{kBq/m}^3)) = 62.5 \text{ min.}$$

That looks pretty but makes no sense because of the longer response time. So we will set the interval to 120 Minutes and ask for the statistical error in this case:

$$N(\text{slow}) = C_{Rn} \cdot T \cdot S = 0.2 \text{ kBq/m}^3 \cdot 120 \text{ min} \cdot 8 \text{ cts}/(\text{min} \cdot \text{Bq/m}^3) = 192 \text{ cts}$$

$$E(10\%) = 100\% \cdot 1 \cdot \frac{100}{N} = 100\% \cdot 1 \cdot \frac{100}{192} = 5.21\%$$

Now one could say 5.21% is not sure enough, I want to choose 20% confidence interval to get a more trustable result:

$$E(20\%) = 100\% \cdot 2 \cdot \frac{100}{N} = 100\% \cdot 2 \cdot \frac{100}{192} = 10.42\%$$

For interpretation look at the begin of this chapter.

Is an observed concentration change statistical significant or not?

If you have a look at the acquired time distribution you will see variations of the concentration from point to point. The question is now: Is it a real change in the Radon concentration or only a statistical fluctuation?

The test is very simple: Define a confidence level with respect to your needs and look at the statistical error bands of the two points of interest. If the error bands do not overlap each other, the change in the Radon concentration is significant otherwise it “can be or not can be”.

Example 1:

Reading 1: 1500 Bq/m³ ± 10% ± error band [1350 ... 1650 Bq/m³]

Reading 2: 1300 Bq/m³ ± 13% ± error band [1131 ... 1469 Bq/m³]

The upper limit of the error band of the reading 2 is higher than the lower limit of the error band of reading 1. Because the “true” value could be placed within 1350 Bq/m³ and 1469 Bq/m³, the variation of both readings is not statistically significant.

Example 2:

Reading 1: 1500 Bq/m³ ± 10% ± error band [1350 ... 1650 Bq/m³]

Reading 2: 1000 Bq/m³ ± 15% ± error band [850 ... 1150 Bq/m³]

The error bands of the readings do not overlap each other. Therefore, a statistically significant concentration change is given.

Two arbitrary points of a measurement series may be considered using this test. It is not necessary that the points are direct neighbours.

Detection Limit

The term Detection Limit defines the smallest value of the Radon concentration which delivers a non zero reading of the instrument within a given integration interval (at least 1

decay per interval). Because of the statistical behaviour a related confidence interval has to be stated.

Why is it necessary to know the Detection Limit? If the set integration interval is short and the Radon concentration low, the expected “true” value of the number of detected decays may be around or less than 1. Because of the statistical variations, intervals without any detected decay will appear frequently. The most extreme situation would be a measurement series with a lot of “zero” intervals and only one interval with one detected decay (because a decay can not be split).

When calculating the Radon concentration by the given formula, the concentration value for the interval with the one count is much too high while all other values show zero. Then, all intervals have to be averaged to get a usable result. This procedure is nothing else than to create an integration interval long enough to meet the Detection limit for the applied Radon concentration. To avoid zero readings, set the integration interval with respect to the lowest expected concentration level during measurement.

The mean („true“) value of the number of decays during an integration interval in case of a Radon concentration in the surrounding of the detection limit is less than 16 and therefore the statistical fluctuations have to be derived by the Poisson distribution. The stated confidence interval gives the probability that the detected number of decays within the interval is not zero.

Confidence Interval	Required Mean Value for N at the Detection Limit
63,2 %	1
95 %	3
99,75 %	6

Example:

Determination of the detection limit of the Monitor using the „Fast-Mode“ and an integration interval of 60 Minutes. The confidence interval shall be 95% (that means in about 95 from 100 intervals a no zero reading should appear):

Required mean value (number of counts from the table): $N = 3$.

Calculating the detection limit by the formula:

$$C = N / (T * S) = 3 \text{ cts} / (60 \text{ min} * 8 \text{ cts}/(\text{min} * \text{kBq}/\text{m}^3)) = 0,00625 \text{ kBq}/\text{m}^3 = 6,25 \text{ Bq}/\text{m}^3$$

The detection limit in this case is $6,25 \text{ Bq}/\text{m}^3$.